Distributed Algorithm Design for Probabilistic Demand Response

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1. INTRODUCTION

One of the major issues with the integration of renewable energy sources into the power grid is the increased uncertainty and variability that they bring [1]. The limited capability to accurately predict this variability makes it challenging for the load serving entities (LSEs) to respond to it [3]. If this variability is not sufficiently addressed, it will limit the further penetration of renewables into the grid and even result in blackouts [4]. Various approaches have been implemented or proposed to address this issue. These include improving renewable generation forecast, aggregating diverse renewable sources, fast-responding reserve generators, energy storage, and demand response (DR), among others. Compared to energy storage, DR has advantages to provide reserves to the LSEs in a cost-effective and environmentally friendly way [11, 16].

DR programs work by changing customers' loads when the power grid experiences a contingency such as a mismatch between supply and demand. The decision that must be made by DR programs is how much load demand each customer should change. Uncertainties from both the customer-side and LSE-side make designing algorithms for DR a major challenge. LSEs make predictions about the net load demand and purchase capacity to dispatch controllable supply accordingly, but they do not know the true mismatch between supply and demand until the time arrives. On the other hand, customers who are accustomed to having electricity supply on demand are not be able to accurately estimate how much disutility a future change in load demand would bring them. These uncertainties mean that the LSE does not know how exactly how much aggregate DR it will need and the customers do not know how much change they will be willing to provide.

Also for large-scale algorithms with many decisions such as DR programs, distributed algorithm design is important. As networks grow, the increased communication overhead required for a centralized decision maker becomes infeasible. More importantly, privacy requirements may not allow a central entity to know the objectives and constraints of all the users. This is especially true in the case of deregulated power markets [8]. Distributing algorithms is a major challenge for DR because the LSE needs information about the customers' decisions.

This paper makes the following main contributions: *First*, we model the social cost minimization problem using stochastic optimization that jointly optimizes DR participation and capacity planning (Section 2). *Second*, we propose a simple contract between customers and the LSE, and design a distributed algorithm to find the optimal contract parameters (Section 3). *Third*, we show that the distributed algorithm converges quickly with a real world trace (Section 4). The major difference in our work as compared to existing work is that we design a contract between each customer and the LSE that adjusts the customers' loads according to real-time system parameters in contrast to robust real-time pricing strategies, e.g [10, 12, 15].

2. MODEL

We consider a two-stage decision problem in which an LSE first purchases capacity to handle an unknown amount of supply-demand mismatch, and second procures a total amount of load reduction from a set of customers $\mathcal{V}$ after the supply-demand mismatch is revealed. Note that we only frame DR as a load reduction for ease of explanation. Our approach is general enough for DR to be framed as any change of load demand. We ignore the power network constraints in this paper. However, our model and algorithms can be extended with extra effort to incorporate those power network constraints.

Customers

Let d_i be the actual power demand and $\hat{d}_i$ be the predicted power demand of customer $i \in \mathcal{V}$ for a particular timeslot. Therefore, we denote $\delta_i := d_i - \hat{d}_i$ as the customer demand's mismatch and model it as a random variable. In real-time, the customer observes its own real power demand $d_i = \hat{d}_i + \delta_i$ in the absence of a DR program and decides x_i under a particular DR program as the amount of demand reduction from d_i. So the actual power consumption under a DR program becomes $d_i - x_i$.

To model the loss of utility caused by the reduction in power consumption x_i from the original demand d_i, we assume there is a cost function $C_i(x_i)$. The function is inherently different for different timeslots and may not be known until at (or just before) the time of consumption. The uncertainty in this function can be modeled by using a random variable that parameterizes $C_i(x_i)$, (e.g. a_i in $C_i(x_i) = a_i x_i^2$). We use the function $\hat{C}_i(x_i)$ to represent its estimated cost function.

Load Serving Entity

We consider the general case where the LSE has volatile renewable energy generation and is responsible for handling any supply-demand mismatch with dispatchable resources. The uncertainty from the renewables and the customers' demands combine to form the aggregate mismatch D which is modeled as a random variable. Therefore, after the customers apply their load reductions, the LSE has $D - \sum_i x_i$ remaining mismatch to manage.

For any remaining mismatch, the LSE has to bear the cost denoted by the penalty function $C_g(D - \sum_i x_i)$. Specifically, this is the cost imposed on the LSE to close the gap through actions such as employing fast responding reserves or grid energy storage.

In order for the LSE to tolerate the mismatch and prevent blackouts, the LSE must purchase long-term energy storage or reserves in some forward market denoted by κ. Denote $C_{\text{cap}}(\kappa)$ as the cost of the energy storage/reserve capacity κ amortized to a single timeslot. This gives us the mismatch constraint

$$-\kappa \le D - \sum_{i\in\mathcal{V}} x_i \le \kappa \tag{1}$$

so that the LSE has the ability to handle any remaining mismatch.

Optimization Problem

The goal is to *simultaneously* decide the capacity planning κ and a practical DR policy $\mathbf{x}(D,\boldsymbol{\delta})$ to minimize the *expected* social cost caused by a random aggregate supply-demand mismatch D (which captures mismatches from both the generation side and the load side).

$$\min_{\kappa,\mathbf{x}(D,\boldsymbol{\delta})} C_{\text{cap}}(\kappa) + \mathbb{E}_{D,\boldsymbol{\delta},C_i(\cdot)}\left[\sum_i C_i(x_i(D,\delta_i)) + C_{\text{g}}\left(D - \sum_i x_i(D,\delta_i)\right)\right]$$

$$\text{s.t.}\ \max_{D,\boldsymbol{\delta}}\left\{D - \sum_i x_i(D,\delta_i)\right\} \le \kappa \tag{2a}$$

$$\min_{D,\boldsymbol{\delta}}\left\{D - \sum_i x_i(D,\delta_i)\right\} \ge -\kappa. \tag{2b}$$

The expectation is taken with respect to the aggregate mismatch D, individual customer mismatches δ_i, and the customer cost functions $C_i(\cdot)$ since they are not known before DR operation. We note that (2a) and (2b) are worst-case constraints so that the remaining mismatch does not go beyond the purchased capacity.

We make the following mild assumptions. We assume that the cost functions are convex. The convexity assumption on the customer-side is consistent with the concavity assumption of customer utility functions as was done in [14]. A simple but widely used example is the quadratic function, i.e., $C_i(x_i) = a_i x_i^2$ [17]. On the LSE-side, $C_g(\cdot)$ can also be a quadratic function [13], and $C_k(\kappa)$ can be linear. Also, the randomness in a customer's cost function $C_i(\cdot)$ and the mismatch D are assumed to both be stationary. This assumption is reasonable since the randomness in D is due to the *prediction error* of the customers' load demands and renewable energy supply.

The two main challenges of Problem (2) are (i) deciding the optimal capacity κ *before* operating the DR policy, and (ii) optimizing an online DR policy.

3. POLICY DESIGN

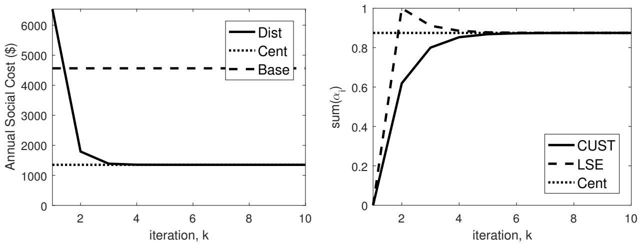
Linear contract

Motivated by the desire to find a simple DR policy $\mathbf{x}(D,\boldsymbol{\delta})$ that preserves convexity and can be decided jointly with capacity, we focus on a simple but powerful linear contract that is a function of the aggregate and individual mismatches:

$$x_i(D,\delta_i) = \alpha_i D + \beta_i \delta_i + \gamma_i \tag{3}$$

which is the optimal form of a DR policy when the cost functions are quadratic as shown in our extended version [9].

This contract combines the global aggregate mismatch D with each customer's local mismatch δ_i to decide what that customer's change in demand should be. Intuitively, there are three components: $\alpha_i D$ implies each customer shares some (predefined) fraction of the global mismatch D; $\beta_i\delta_i$ means customer i may need to take additional responsibility for the mismatch due to his own demand fluctuation and estimation error; finally, γ_i, the constant part, can help when the random variables $\mathbb{E}[D]$ and/or $\mathbb{E}[\delta_i]$ is nonzero. Then the LSE needs to solve (2) with (3) to obtain the optimal parameters for the linear contract, i.e., $\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma}$, as well as the optimal capacity κ. We have the following theorem proved in [9]:



(a) Social Cost (b) Total Reactive DR

Figure 1: Convergence of the distributed algorithm. Baseline is to optimize DR after purchasing enough capacity to close the mismatch.

THEOREM 1. *Problem* (2) *with the linear contract* (3) *is a convex optimization problem.*

Distributed algorithm

In most cases, the LSE's information on the customers' cost functions is much less accurate than the customers themselves'. This can also be due to privacy concerns. To handle this, we design a distributed algorithm so that the LSE does not need the information of the customer cost functions.

First, we introduce and substitute (u_i, v_i, w_i) as the customer's copy of $(\alpha_i,\beta_i,\gamma_i)$ in each of their estimated cost functions $\hat{C}_i(\cdot)$ to get

$$\min_{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma},\mathbf{u},\mathbf{v},\mathbf{w},\kappa} C_{\text{cap}}(\kappa) + \sum_{i\in\mathcal{V}} \mathbb{E}_{\delta,\delta_r}\left[\hat{C}_i(u_i D + v_i\delta_i + w_i)\right] + \mathbb{E}_{\delta,\delta_r}\left[C_{\text{g}}\left(\sum_{i\in\mathcal{V}}(D - \sum_{i\in\mathcal{V}}(\alpha_i D + \beta_i\delta_i + \gamma_i)\right)\right]$$

$$\text{s.t.}\ (2a),(2b) \tag{4a}$$

$$u_i = \alpha_i,\quad v_i = \beta_i,\quad w_i = \gamma_i,\quad i\in\mathcal{V} \tag{4b}$$

Problem (4) can be split where each customer controls its own (u_i, v_i, w_i) and the LSE controls $(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma})$ by using dual decomposition of constraint (4b). Let (π_i,λ_i,μ_i) be the dual prices for each customer corresponding to constraint (4b). Therefore $\pi_i u_i + \lambda_i v_i + \mu_i w_i$ is the total payment to customer i for following the linear demand response contract. Accordingly, (4) is decomposed into the individual customer optimization problem

$$\min_{u_i,v_i,w_i} \mathbb{E}_{D,\delta_i}\left[\hat{C}_i(u_i D + v_i\delta_i + w_i)\right] - \pi_i u_i - \lambda_i v_i - \mu_i w_i, \tag{5}$$

and the LSE's optimization problem among all the customers

$$\min_{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma},\kappa} C_{\text{cap}}(\kappa) + \sum_{i\in\mathcal{V}}(\pi_i\alpha_i + \lambda_i\beta_i + \mu_i\gamma_i) + \mathbb{E}_{D,\boldsymbol{\delta}}\left[C_{\text{g}}\left(D - \sum_{i\in\mathcal{V}}(\alpha_i D + \beta_i\delta_i + \gamma_i)\right)\right] \tag{6}$$

$$\text{s.t.}\ (2a),(2b).$$

Problems (5) and (6) can be solved with standard stochastic optimization techniques such as the Stochastic Subgradient Method with Monte Carlo sampling [7]. To solve the decomposed problems, we must ensure the customers' and LSE's decisions satisfy (4b). We achieve this by applying the Subgradient Method (see [6] Chapter 6) to obtain the optimal dual prices in the following

Distributed Algorithm:

0. **Initialization:** $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \mathbf{u}, \mathbf{v}, \mathbf{w}, \boldsymbol{\pi}, \boldsymbol{\lambda}, \boldsymbol{\mu}) := \mathbf{0}$.
1. **LSE:** receives (u_i, v_i, w_i) from each customer $i \in \mathcal{V}$.
   - Solves Problem (6) and updates $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$ with the optimal solution.
   - Updates the stepsize:
   $$\eta = \frac{\zeta/k}{||(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}) - (\mathbf{u}, \mathbf{v}, \mathbf{w})||_2} \quad (7)$$
   where ζ is a small constant and k is the iteration number.
   - Updates the dual prices, $\forall i \in \mathcal{V}$:
   $$(\pi_i, \lambda_i, \mu_i) := (\pi_i, \lambda_i, \mu_i) + \eta\left((\alpha_i, \beta_i, \gamma_i) - (u_i, v_i, w_i)\right) \quad (8)$$
   - Sends $(\pi_i, \lambda_i, \mu_i)$ to the each customer respectively.
2. **Customer** $i \in \mathcal{V}$**:** receives $(\pi_i, \lambda_i, \mu_i)$ from LSE.
   - Solves Problem (5) and updates (u_i, v_i, w_i) with optimal solution.
   - Sends (u_i, v_i, w_i) to the LSE.
3. Repeat Steps 1-2 until $||(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}) - (\mathbf{u}, \mathbf{v}, \mathbf{w})||_2 \leq \epsilon$ where ϵ is the tolerance on magnitude of the subgradient.

When the LSE signals the customers for DR and they respond accordingly, each customer is paid $\pi_i u_i + \lambda_i v_i + \mu_i w_i$ by the LSE. We now establish the convergence and optimality for the proposed distributed algorithm proved in [9]:

THEOREM 2. *The distributed algorithm's best dual prices converge to the optimal dual prices of Problem* (4)*.*

4. PERFORMANCE EVALUATION

We aim to use realistic parameters in the experimental setup to evaluate the convergence of our distributed algorithm. We model an LSE supplying power to 300 customers for a demand response timeslot that is five minutes long and observe its social cost over a year long operation. The LSE must first purchase capacity for which we model the cost as a linear function $c\kappa$ with a cost parameter $c = \$1$/kW-mo. The generation cost function for the LSE is modeled as a quadratic function $A\left(D - \sum_i x_i\right)^2$ with the parameter $A = \$0.01/12^2/\text{kW}^2$. For this cost function setting, a deviation of 60kW for five minutes is equivalent to an energy cost of \$0.05/kWh and matches the intuition that larger mismatches are increasingly more expensive to manage. Each customer has a particular demand of load. To model this we utilize the traces obtained from the UMass Trace Repository which give very granular load measurements from three homes [5]. We model the cost incurred by each customer to change its consumption as a quadratic cost $a_i x_i^2$ with the parameter $a_i \in \$[1, 10]/12^2\text{kW}^2$. Under these settings, a consumption decrease of 0.3kW for five minutes would cost the customer an energy price equivalent to \$0.025-0.25/kWh. To generate customer cost uncertainties we randomly choose $\hat{a}_i$ from a bounded normal distribution for each customer's estimated cost function. Renewable generation is incorporated into our simulations by using the ISO-NE's data on hourly wind power production for the same dates as the UMass data [2]. The amount of wind capacity is scaled to 100kW. The historical data sets for each customer were generated from the available trace data (Homes A,B,C and ISO-NE wind production). They were made by bootstrapping 100 customers from each of the UMass Homes A/B/C. We also do this for the ISO-NE wind data which is first normalized by the maximum power output so that we can scale wind power accordingly. Sampling from these historical sets is how the expectation is evaluated in Equations (5) and (6).

Convergence of the distributed algorithm.

We consider the convergence of our distributed algorithm. Figure 1(a) illustrates that the social cost of the distributed algorithm converges quickly to that of the centralized algorithm and Figure 1(b) gives the trajectory of the total fraction of aggregate mismatch absorbed by all of the customers. It validates the convergence analysis for the distributed algorithm. For the parameters, even if we start with $\alpha_i = 0 : \forall i \in \mathcal{V}$, it quickly converges to the optimal α_i and stays there.

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