AMS-528 Numerical Analysis (III)

Assignment 5

Due: April 23th

Please post all movies on your webpage, send the address to the TA and copy me. Thank you!

(1). For a system of linear wave equations

\[ V_t + AV_x = 0 \]

where

\[ V = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \quad A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \]

Write the upwind and Lax-Wendroff schemes for this system of equations. Solve the system of equation with initial condition

\[ V(x, 0) = \begin{pmatrix} \sin \pi x \\ \cos \frac{1}{2} \pi x \\ -\sin 2\pi x \end{pmatrix} \quad \text{if} \quad -1 < x < 1 \]

and

\[ V(x, 0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{otherwise} \]

Solve the equation in the interval \( x \in [-2, 6] \) with \( \Delta x = 0.01 \) and \( \max(R) = 0.9 \) to \( t = 2 \). Show movies of \( V \) with frame interval \( \Delta t_m = 0.1 \).

(2). Solve the following problem using (a) Lax-Friedrichs, (b) Lax-Wendroff and (c) Godunov schemes with exact Riemann solution.

\[ v_t + v v_x = 0 \]

\[ v(x, 0) = \begin{cases} 
\cos \pi x + 1.2 & \text{if } -1 < x < 1 \\
0.2 & \text{otherwise} 
\end{cases} \]

\[ v_x(-2, t) = v_x(2, t) = 0 \]
Use $\Delta x = 0.01$ (200 grid points) to produce animated plot (movie) of the solution in $x \in [-2, 2]$ to $t = 1.0$. Create movies for each scheme with movie frame interval $\Delta t_{\text{frame}} = 0.1$. Using Godunov schemes with 3200 grid points as the “exact” solution, compute the $L_1$, $L_2$ and $L_\infty$ error norms at $t = 1.0$ for 200, 400, 800, 1600 grid points.