(1). Solve

\[ v_t + v_x = 0, \quad x \in [0, 6], \quad t > 0 \]

with the initial condition

\[ v(x, 0) = \begin{cases} \sin^2 \pi(x - 1) & \text{if } 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases} \]

and boundary condition

\[ v(0, t) = v(6, t). \]

Find the solution from \( t = 0 \) to \( t = 4 \), using (a) backward difference (upwind), (b) Lax-Friedrich, (c) Lax-Wendroff, and (d) MacCormack schemes. Use number of mesh intervals \( N = 200 \) (201 grid points) and \( R = \frac{\Delta t}{\Delta x} = 0.5 \). Compare and contrast your results with the exact solution at \( t = 4 \).

(2). Compare the numerical solutions with the exact solution at \( t = 4 \). Tabulate \( L_1 \), \( L_2 \) and \( L_\infty \) norms of the solution errors for each scheme of problem (1) in the sequences of mesh refinement \( N = 200, N = 400, N = 800, N = 1600 \). The table should look like the following:

\[
\begin{array}{|c|c|c|c|c|}
\hline
N & \text{Upwind} & \text{Lax-Friedrich} & \text{Lax-Wendroff} & \text{MacCormack} \\
\hline
200 & & & & \\
400 & & & & \\
800 & & & & \\
1600 & & & & \\
\hline
\end{array}
\]
(3). Solve the same problem in (1) using implicit central difference, and Crank-Nicolson scheme. Use $R = 0.5$ and $R = 1.5$. Use number of mesh blocks $N = 200, 400, 800, 1600$ respectively. Plot the solution at $t=4$. 

<table>
<thead>
<tr>
<th>$N$</th>
<th>Upwind</th>
<th>Lax-Friedrich</th>
<th>Lax-Wendroff</th>
<th>MacCormack</th>
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<tbody>
<tr>
<td>200</td>
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$L_\infty$ Error Norm