AMS-528 Numerical Analysis (III)

Assignment 1

Due: Feb. 12th

(1). Solve the following problem

\[ v_t = \nu v_{xx}, x \in (0, 1), t > 0, \nu = 1/6 \]

\[ v(x, 0) = \sin 4\pi x \]

\[ v(0, t) = 0, v(1, t) = 0, t \geq 0 \]

using the scheme

\[ \frac{u_{k+1}^n - u_k^n}{\Delta t} = \nu \frac{u_{k+1}^n - 2u_k^n + u_{k-1}^n}{\Delta x^2} \]

Choose \( \Delta x = 0.1, \Delta t = 0.01 \).

Find the solutions at \( t = 0.06, 0.1, 0.9, 10.0 \). Compare and contrast your solution with the exact solution (using graphics).

(2). Using the same scheme to solve the same problem in (1). Report the results at time \( t = 0.06 \) and \( t = 0.1 \).

Use (a) \( \Delta x = 0.1, \Delta t = 0.02 \); (b) \( \Delta x = 0.05, \Delta t = 0.005 \); (c) \( \Delta x = 0.01, \Delta t = 0.0002 \)

Compare and contrast your results.

(3). Solve the problem in (1) using the leapfrog scheme

\[ \frac{u_{k+1}^n - u_k^{n-1}}{2\Delta t} = \nu \frac{u_{k+1}^n - 2u_k^n + u_{k-1}^n}{\Delta x^2} \]

Using \( \Delta x = 0.1, \Delta t = 0.02 \). Find the solutions at \( t = 0.06, 0.1, 0.9 \). Use the exact solution at \( \Delta t \) to get started.