AMS 210 (Applied Linear Algebra)  
HW # 3  
Section 1.3:

6.  
(b) Vector equation:  
\[(x, y, z) = (1, 2, 3) + t(-2, -4, -6)\]
Parametric equations:  
\[x = 1 - 2t, \ y = 2 - 4t, \ z = 3 - 6t\]

9. A point-normal form is \[3(x + 1) + 2(y + 1) + (z + 1) = 0\]. The corresponding general equation is \[3x + 2y + z = -6\].

11. If \(P, Q, R\) are the points \(P(1, 1, 4), Q(2, -3, 1), R(3, 5, -2)\), then the vectors \(\overrightarrow{PQ} = (1, -4, -3)\) and \(\overrightarrow{PR} = (2, 4, -6)\) are parallel to the plane through \(P, Q\) and \(R\). Thus a vector equation for the plane, expressed in component form, is

21. Any plane with general equation of the form \[3x + 2y - z = k\] has \(n = (3, 2, -1)\) as a normal vector, and thus is parallel to \(3x + 2y - z = 1\). In order for such a plane to pass through the point \(P(1, 1, 1)\) we must have \(k = 4\). Thus the general equation of the plane is \[3x + 2y - z = 4\]. Finally, we can get parametric equations for the plane by (for example) solving the equation for \(z\) in terms of \(x\) and \(y\), and then making \(x\) and \(y\) into parameters. This leads to:
\[x = t_1, \ y = t_2, \ z = 3t_1 + 2t_2 - 4\]

25. The plane \(x + y + z = 0\) has normal vector \(n = (1, 1, 1)\); thus any line perpendicular to the plane must be parallel to \(n\). Parametric equations for the line that passes through the point \(P(2, 0, 1)\) and is parallel to \(n = (1, 1, 1)\) are
\[x = 2 + t, \ y = t, \ z = 1 + t\]

Section 2.1

10  
(c) \(x_1 = r, x_2 = s, x_3 = t, x_4 = 20 - 4r - 2s - 3t\), where \(-\infty < r, s, t < \infty\).

15. If \(k \neq 6\), then the equations \(x - y = 3, 2x - 2y = k\) represent nonintersecting parallel lines and so the system of equations has no solution. If \(k = 6\), the two lines coincide and so there are infinitely many solutions: \(x = 3 + t, y = t\), where \(-\infty < t < \infty\).

19. The augmented matrix of the system is
\[
\begin{bmatrix}
1 & 2 & 0 & -1 & 1 \\
0 & 3 & 2 & 0 & -1 \\
0 & 3 & 1 & 7 & 0 \\
\end{bmatrix}
\]

31. (a) \[3c_1 + c_2 + 2c_3 - c_4 = 5\]
\[c_2 + 3c_3 + 2c_4 = 6\]
\[-c_1 + c_2 + 5c_4 = 5\]
\[2c_1 + c_2 + 2c_3 = 5\]