6. The reduced row echelon form for $A$ is

$$
\begin{bmatrix}
1 & 0 & \frac{2}{3} & \frac{3}{3} & \frac{4}{3} & -\frac{5}{3} \\
0 & 1 & -\frac{1}{3} & -\frac{4}{3} & \frac{1}{3} & \frac{4}{3} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Thus $\text{rank}(A) = 2$, and a general solution of $Ax = 0$ is given by

$$
x = r\left(-\frac{1}{3}, -\frac{1}{3}, 1, 0, 0, 0\right) + s\left(-\frac{5}{3}, \frac{7}{3}, 0, 1, 0, 0\right) + t\left(-\frac{2}{3}, \frac{4}{3}, 0, 0, 1, 0\right) \\
+ u\left(-\frac{4}{3}, -\frac{1}{3}, 0, 0, 0, 1\right) + v\left(\frac{5}{3}, -\frac{4}{3}, 0, 0, 0, 0\right)
$$

It follows that $\text{nullity}(A) = 5$. Thus $\text{rank}(A) + \text{nullity}(A) = 2 + 5 = 7$.

Sec. 8.2

10. The characteristic polynomial of $A$ is $(\lambda + 1)(\lambda - 3)^2$. Thus the eigenvalues of $A$ are $\lambda = -1$ and $\lambda = 3$, with algebraic multiplicities 1 and 2 respectively. The eigenspace corresponding to $\lambda = -1$ is 1-dimensional, and so $\lambda = -1$ has geometric multiplicity 1. The eigenspace corresponding to $\lambda = 3$ is the solution space of the system $(3I - A)x = 0$ which is

$$
\begin{bmatrix}
2 & 2 & 0 \\
2 & 2 & 0 \\
-2 & -2 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
$$

The general solution of this system is $x = s\begin{bmatrix}-1 \\
1 \\
0
\end{bmatrix} + t\begin{bmatrix}0 \\
0 \\
1
\end{bmatrix}$; thus the eigenspace is 2-dimensional, and so $\lambda = 3$ geometric multiplicity 2.

19. The characteristic polynomial of $A$ is $p(\lambda) = \lambda^3 - 6\lambda^2 + 11\lambda - 6 = (\lambda - 1)(\lambda - 2)(\lambda - 3)$; thus $A$ has three distinct eigenvalues $\lambda_1 = 1$, $\lambda_2 = 2$ and $\lambda_3 = 3$. Corresponding eigenvectors are $v_1 = \begin{bmatrix}1 \\
1 \\
1
\end{bmatrix}$, $v_2 = \begin{bmatrix}2 \\
3 \\
3
\end{bmatrix}$, and $v_3 = \begin{bmatrix}1 \\
1 \\
4
\end{bmatrix}$. Thus $A$ is diagonalizable and $P = \begin{bmatrix}1 & 2 & 1 \\
1 & 3 & 3 \\
1 & 3 & 4
\end{bmatrix}$ has the property that

$$
P^{-1}AP = 
\begin{bmatrix}
3 & -5 & 3 \\
-1 & 3 & -2 \\
0 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
-1 & 4 & -2 \\
-3 & 4 & 0 \\
-3 & 1 & 3
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 3
\end{bmatrix}
$$

Note. The diagonalizing matrix $P$ is not unique; it depends on the choice (and the order) of the eigenvectors. This is just one possibility.