Chapter 1: Making Decisions

Hypothesis:
- Null hypothesis (H₀)
- Alternative hypothesis (H₁)

Statistically Significant → Reject H₀

Error:
- Type I error (H₀ is true, reject H₀)
- Type II error (H₁ is true, accept H₀)

α = Level of significance
  = chance of Type I error occurrence
  = chance of rejecting H₀ when it is true

β = chance of Type II error occurrence
  = chance of accepting H₀ when H₁ is true

Direction of extreme:
- One sided (to the right, to the left)
- Two sided
Decision rule \( \rightarrow \) significance level \( \alpha \)

significant level \( \alpha \) \( \rightarrow \) decision rule

How usual are the data?

\[ p \text{-value} \]

Definition: (probability)

The \( p \)-value is the chance, computed under the assumption that \( H_0 \) is true, of getting the observed value plus the chance of getting all \( \alpha \) more extreme values.

\[
\begin{cases}
H_0 \text{ is true} \\
\text{direction of extreme}
\end{cases}
\]

A small \( p \)-value indicates that the observed value and even more extreme values are unlikely if \( H_0 \) is true. \( \rightarrow \) reject \( H_0 \)

- showing stronger evidence against \( H_0 \).
Ex: What is in the bag?

Bag A: $-1000, more $10, $20.
Bag B: $1000, more $50, $60.

H₀: The shown bag is Bag A.
H₁: The shown bag is Bag B.

We got a $30 from the shown bag.

\[ \text{p-value} = \begin{cases} \text{chance of getting$30 +} \\ \text{chance of getting$40, $50, $60, $1000 from Bag A.} \end{cases} \]

\[ = \frac{6}{20} = 0.3 \]

2) $40 observed.

\[ \text{p-value} = \begin{cases} \text{chance of getting$40 +} \\ \text{chance of getting$50, $60, $1000 from Bag A.} \end{cases} \]

\[ = \frac{4}{20} = 0.2 \]
\( \alpha \), p-value

Probabilities can be compared.

Approach: Compare p-value to the required significance level \( \alpha \), and make decisions.

(Classical approach: decision rule \( \rightarrow \alpha 

P-value approach: compare p-value to required \( \alpha \).

\begin{align*}
\text{P-value approach:} \\
\text{If } p\text{-value} &\leq \alpha \rightarrow \text{the data are statistically significant at given level } \alpha. \\
&\Rightarrow \text{reject } H_0. \\
\text{If } p\text{-value} &> \alpha \rightarrow \text{the data are not statistically significant at given } \alpha. \\
&\Rightarrow \text{accept } H_0.
\end{align*}

Ex: What's in the bag?

- required \( \alpha = 0.1 \\
- 36$ observed from the shown bag.

\[
p\text{-value} = 0.3 > \alpha \Rightarrow \text{accept } H_0 \text{ (shown bag } 5 \text{'s bag)}
\]
(3) $60 observed
\[ p\text{-value} = \text{the chance of getting } \$60 \]
\[ + \text{chance of getting } \$1000 \text{ from Bag A} \]
\[ = \frac{1}{20} = 0.05 < \alpha \]
\[ \Rightarrow \text{data are statistically significant} \]
\[ \Rightarrow \text{reject } H_0 \text{ (shown bag is bag B).} \]

Let's do it.

<table>
<thead>
<tr>
<th>Study</th>
<th>$H_0$</th>
<th>$H_1$</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\geq 54\ m$</td>
<td>$&lt; 54\ m$</td>
<td>0.0251</td>
</tr>
<tr>
<td>B</td>
<td>=</td>
<td>$\neq$</td>
<td>0.0018</td>
</tr>
<tr>
<td>C</td>
<td>$\leq 0.33\ m$</td>
<td>$&gt; 0.33\ m$</td>
<td>0.3590</td>
</tr>
</tbody>
</table>

(a) More support for $H_0$?
Longer $p$-value $\uparrow$ Study C

(b) accept $H_1$, reject $H_0$
When $H_0$ is true.
\[ \Rightarrow \text{Type I error} \]

(c) Study C: "not statistically significant"
\[ \Rightarrow \text{accept } H_0. \]
Chapter 9  Making decisions about a population proportion with confidence

Overall procedure:

1. Competing theories (H₀, H₁)
2. Collect data
3. Summarize (p-value)
4. Interpret, make decision

Population proportion \( P \)
Sample proportion \( \hat{P} \)

\[ \text{Parameter: } P \quad \text{Statistic: } \hat{P} \]

Review:
Sampling distribution of \( \hat{P} \):
\[ \mu_{\hat{P}} = P, \quad \sigma_{\hat{P}} = \sqrt{\frac{P(1-P)}{n}} \]

If "sufficiently" large sample \((np \geq 5 \text{ and } n(1-p) \geq 5)\):
\( \hat{P} \approx N(P, \sqrt{\frac{P(1-P)}{n}}) \).

9.3 Testing hypothesis about a population proportion.
A one sided to the left.
B two sided
C one sided to the right.
(focusing on B)
Underlying procedure for hypothesis testing:

1. State the population and corresponding parameter of interest.

2. State the competing theories (H₀, H₁).
   (Statements about the population, not by the sample, expressed in terms of the parameter)

3. State the significance level α for the test.

4. Collect and examine the data.

5. Compute a test statistic using the data and determine the corresponding p-value.

6. Make decision, state a conclusion.

Let's do it with some examples:

(a). H₀: \( p = 0.5 \) vs. H₁: \( p > 0.5 \)
   (\( p \) = proportion of pregnancies not planned)

(b). H₀: \( p = 0.03 \) vs. H₁: \( p < 0.03 \)
   (to the left)

(c). H₀: \( p = 0.25 \) vs. H₁: \( p \neq 0.25 \)
   (two sided)