Random variable \{ discrete
\}

- Mass function
- Density function

Binomial Random Variable

\{ n \text{ independent Bernoulli trials} \}

- Number of success = \( X \)
- \( P(\text{success}) = p \)
- \( P(\text{failure}) = 1 - p = q \)

\( X \sim \text{Bin}(n, p) \quad n, \ p \)
Probability distribution of a binomial random variable

\[ p(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \]

\( \binom{n}{x} \) "n choose x" = \( \frac{n!}{(n-x)! x!} \)

- combinations

\[ x = 0, 1, 2, \ldots, n \]

Mean, variance and STD

Mean (expected value)

\[ E(x) = \mu = np \]

Variance:

\[ \text{Var}(x) = \sigma^2 = npq = np(1-p) \]

STD:

\[ SD(x) = \sigma = \sqrt{npq} \]
Ex (7.60) Binomial Random Variable $X$. 
$X \sim \text{Bin}(10, 0.4)$. 
$p = 0.4$, \quad q = 0.6$

1. $P(X = 2)$

$P(x) = P(X = x) = \binom{n}{x} p^x q^{n-x}$

$[n = 2, \quad n = 10, \quad p = 0.4, \quad q = 0.6]$.

$P(X = 2) = \binom{10}{2} (0.4)^2 (0.6)^8 = 0.12093$

2. $P(X > 8) = P(X = 9) + P(X = 10)$

$= \binom{10}{9} (0.4)^9 (0.6)^1 + \binom{10}{10} (0.4)^{10} (0.6)^0$

$= 0.00157 + 0.000015 = 0.00168$

3. $P(X \leq 7) = 1 - P(X > 7)$

$= 1 - (P(X = 8) + P(X = 9) + P(X = 10))$

$= 0.99837$. 

4. $P(X = 1.5) = 0$
5. \[ E(X) = np = 10 \times 0.4 = 4 \]
6. \[ \text{Var}(X) = npq = 10 \times 0.4 \times 0.6 = 2.4 \]
   \[ \text{SD}(X) = \sqrt{\text{Var}(X)} = \sqrt{2.4} \]

7.5.3 Continuous Random Variable

Probability distribution of a continuous random variable, \( X \) is a function, denoted by \( f(x) \), such that

\[ P(X \text{ takes on values in a set } A) = \text{the area under the function } f(x) \text{ above the set } A. \]
Example (7.68)

\[ X = \text{time for placing a call} \]

- mean = 20 s
- STD = 5 s

\[ X \sim N(\mu, \sigma) \]

\[ X \sim N(20, 5) \]

1. \[ P(X < 10) = P(z < \frac{X - \mu}{\sigma}) = P(z < -2) = 0.0228 \]

2. \[ P(X > 32) = P(z > 2.4) = 1 - P(z \leq 2.4) = 1 - 0.9818 = 0.0182 \]
Basic Probability Rules:

1. \( 0 \leq P(A) \leq 1 \)
2. \( P(\Omega) = 1 \)
3. \( P(A^c) = 1 - P(A) \) \( \text{Complement rule} \)
   \[ P(A) + P(A^c) = 1 \]
4. Addition Rule
   \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
   Disjoint events: \( P(A \cap B) = 0 \)
   \( A \cap B = \emptyset \)
   \( \Rightarrow P(A \cup B) = P(A) + P(B) \)
5. Conditional probability
   \[ P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{whenever} \quad P(B) > 0 \]
   \( \text{Multiplication rule} \)
   \[ P(A \cap B) = P(A|B)P(B) = P(B|A)P(A) \]
Independent event:

If $P(A|B) = P(A)$ or $P(B|A) = P(B)$
A, B are independent events.

$\Rightarrow P(A \cap B) = P(A)P(B)$

6. Partition Rule

$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2)$
If $B_1, B_2$ form a partition of $S$.

7. Bayes's Rule

$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A)}$

$= \frac{P(A|B_1)P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2)}$