Conditional Probability

\[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} \quad \text{where} \quad P(B) > 0 \]

\[ P(A \cap B) = P(A \mid B) P(B) = P(B \mid A) P(A) \]

**Independent events**

\[ P(A \mid B) = P(A) \]
\[ P(B \mid A) = P(B) \]
\[ P(A \cap B) = P(A) P(B) \]

**Disjoint events**

\[ P(A \cup B) = P(A) + P(B) \]

Ex (7.23)

<table>
<thead>
<tr>
<th>Birth Order</th>
<th>0th child</th>
<th>In Between</th>
<th>Youngest</th>
<th>Only child</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>20</td>
<td>30</td>
<td>35</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>310</td>
<td>215</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td>170</td>
<td>340</td>
<td>260</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>110</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1160</td>
</tr>
</tbody>
</table>
1) \[ P(D) = \frac{110}{1160} \]
\[ P(D^c) = 1 - P(D) \]

2) O - Oldest. D - Délinquant
\[ P(D|O) = \frac{P(D \cap O)}{P(O)} = \frac{20}{470} \]
\[ P(D \cap O) = \frac{20}{1160} \]
\[ P(O) = \frac{470}{1160} \]
\[ P(D) = \frac{110}{1160} \]

3) Independent? \[ P(A \cap B) = P(A)P(B)? \]
\[ P(D \cap O) = P(D)P(O)? \]
\[ \frac{20}{1160} \neq \frac{110}{1160} \times \frac{470}{1160} \]
\[ \Rightarrow \text{ Dependent event} \]

Disjoint & Independent
Disjoint, set property.
Independent: probability property.
### Disjoint vs. Independent

<table>
<thead>
<tr>
<th></th>
<th>Disjoint</th>
<th>Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(A \cap B) = 0 )</td>
<td>( P(A \cap B) = P(A)P(B) )</td>
<td></td>
</tr>
<tr>
<td>( P(A \cup B) = P(A) + P(B) )</td>
<td>( P(A \cup B) = P(A) + P(B) - P(A \cap B) )</td>
<td></td>
</tr>
<tr>
<td>( P(A</td>
<td>B) = 0 )</td>
<td>( P(A</td>
</tr>
</tbody>
</table>

#### 7.4.3 Partitioning & Bayes's Rule

The events \( B_1, B_2, \ldots, B_I \) form a partition of the sample space \( S \) if:

1. \( B_1, B_2, \ldots, B_I \) are mutually exclusive.
2. The union of \( B_1, B_2, \ldots, B_I \) is \( S \).

![Partition of Sample Space](image.png)
Ex (7.43) I, II, III

\[ P(I) = 0.7, \ P(II) = 0.2, \ P(III) = 0.1 \]

\[ P(R | I) = 0.03, \ P(R | II) = 0.04 \]

\[ P(R | III) = 0.05 \]

I, II, III form a partition of S.

1. \[ P(R) = P(R | I)P(I) + P(R | II)P(II) + P(R | III)P(III) \]
   \[ = 0.03 \times 0.7 + 0.04 \times 0.2 + 0.1 \times 0.05 = 0.034 \]

2. \[ P(III | R) = \frac{P(R | III)P(III)}{P(R)} \]
   \[ = \frac{0.05 \times 0.1}{0.034} = 0.147 \]

3. \[ P(R)^2 = (0.034)^2 \]

Ex (7.44) +, − D, ND

\[ P(+ | D) = 0.98, \ P(- | ND) = 0.95 \]

\[ P(D) = 0.01, \ P(ND) = 0.99 \]
1. \[ P(+) = P(+|D)P(D) + P(+|\neg D)P(\neg D) \]
\[ P(+|\neg D) = 1 - P(-|\neg D) = 0.05 \]
\[ P(+) = 0.98 \times 0.01 + 0.05 \times 0.99 \]
\[ = 0.0593 \]

2. \[ P(D|+) = \frac{P(+|D)P(D)}{P(+|D)P(D) + P(+|\neg D)P(\neg D)} \]
\[ = \frac{0.98 \times 0.01}{0.0593} \]
\[ = 0.1653 \]