Chapter 6 Using Models to make decisions

Normal Distribution

\[ X \sim N(\mu, \sigma) \]

\[ Z = \frac{X - \mu}{\sigma} \sim N(0, 1) \]

standard normal distribution

\( z \)-score/standard score

area.

Table II

1. Convert to standard score
2. Table II for \( z \)
Find pth percentile

Table III

1. pth percentile for \( z \)
2. \( x = \mu + z \sigma \)
   \[ \text{tpth percentile for } X \sim \mathcal{N}(\mu, \sigma) \]

68-95-99.7 Rule

68\% fall within \([\mu - \sigma, \mu + \sigma]\)
95\% \([\mu - 2\sigma, \mu + 2\sigma]\)
99.7\% \([\mu - 3\sigma, \mu + 3\sigma]\)

6.3.2 Uniform Distribution

\( X \sim \mathcal{U}(a, b) \)

Density

Mean = Median

\[ \frac{a+b}{2} \]
Ex: The travel time for John, a college student, between his home and college is uniformly distributed between 40 and 70 minutes.

\[ X \sim U(40, 70) \]

(a) Sketch the distribution (density curve):

(b) Leaves one hour before class starts. What proportion of days will John be late to class?

\[ X \geq 60 \]

\[ \frac{1}{30} \times 10 = \frac{1}{3} = 33.3\% \]
(c). Find IQR for this distribution

\[
\text{IQR} = Q_3 - Q_1.
\]

\[
\frac{1}{30} (Q_1 - 40) = 0.25 \quad \frac{1}{30} (Q_3 - 40) = 0.75
\]

\[
\Rightarrow Q_1 = 47.5 \quad \Rightarrow Q_3 = 62.5
\]

\[
\text{IQR} = Q_3 - Q_1 = 15
\]

6.4 Modeling discrete variable

List values along with the proportion.

**Mass function** is a model for discrete variable. For each possible value the mass function gives the proportion of units in the population having that value.
1. the values of mass function must be between 0 and 1 and add up to 1.

2. Proportions are measured exactly as the values of function, not as areas under the function.

Probability (Mass function)

\[ P(X) = \text{proportion of units in population having value equal to } X. \]

3. Sum of proportions = 1.0

4. \(0 < P(X) < 1\).

Ex.: Let \(Y\) be number of televisions in a randomly selected Zone City household. The mass function of \(Y\) is given in the following table.
sketch the graph of mass function

\[
P(Y) = \begin{cases} 
0.5 & Y = 0 \\
0.4 & Y = 1 \\
0.3 & Y = 2 \\
0.1 & Y = 3 \\
0.1 & Y = 4 
\end{cases}
\]

\( \mathbb{R} \)

6. What proportion of households have at least one TV?

\[
P(Y \geq 1) = P(1) + P(2) + P(3) + P(4) \\
= 0.3 + 0.4 + 0.2 + 0.05 = 0.95
\]

\[
1 - P(Y < 1) = 1 - P(0) = 1 - 0.05
\]

= 0.95