Midterm 1
2/22, Wed

Chapter 6 Using Models to make decisions

Model

A density function is a (nonnegative) function or curve that describes the overall shape of a distribution. The total area under the curve is equal to 1.
Area under the density function to the left of $X$ measures the proportion of values less than $X$.

**Notation:**

- $\mu$ - mean (population)
- $\sigma$ - STD (population)

### 6.3.1 Normal Distributions

General notation:

\[ X \sim N(\mu, \sigma) \]

- $N(0,1)$
- $N(3,2)$

![Diagram of normal distributions with points of inflection and standard deviations marked]
$X \sim N(\mu, \sigma)$

$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$

\[ z = \frac{x - \mu}{\sigma} \geq \text{number of standard deviations that } x \text{ differs from the mean} \]

$z = -1$

Finding properties for the standard normal distribution.

Table II gives the area to the left of $z$.

$z = 1.22$

0.8888

88.88%
\[ z = -1.08 \quad (0.1401) \]

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\[ \text{Propotion of } z < z < z_2 \]

\[ \text{Ex: } -1.5 < z < 1.3, \quad 83.67\% \]

\[ z = -1.5 \quad (0.0668) \]

\[ z = 1.3 \quad (0.9032) \]

\[ 0.9032 - 0.0668 = 0.8364 \]

The 68-95-99.7 Rule for \((N(\mu, \sigma))\):

1. 68% of the observation fall within \([\mu - \sigma, \mu + \sigma]\).
② 95% fall within \([\mu - 2\sigma, \mu + 2\sigma]\)
③ 99.7% fall within \([\mu - 3\sigma, \mu + 3\sigma]\).

Finding probability for \(N(\mu, \sigma)\)
1. Draw picture \(N(\mu, \sigma)\)
2. Convert to standard score \(z\)
3. Draw picture \(N(0, 1)\)
4. Read off Table II.

Ex: Suppose height in adult males is normally distributed with \(\mu = 70\) inches and \(\sigma = 4\) inches.
\(N(70, 4)\)
(1). The proportion of males between 70 and 82 inches, $70 < h < 82$

$z = \frac{h - \mu}{\sigma} \quad h = 70, \quad z = 0$

$h = 82, \quad z = \frac{82 - 70}{4} = 3$

$0 < z < 3$

$z = 0 (0.5)$

$z = 3 (0.9987)$

$0.9987 \quad 49.87\%$

(2). The proportion of males who are higher than 75 inches, $h > 75$. $h = 75, \quad z = \frac{75 - 70}{4} = 1.25 \quad (0.8944)$

$z > 1.25$

$1 - 0.8944 = 0.1056 \quad (0.56\%)$

Finding the pth percentile for $N(\mu, \sigma)$

1. Find the pth percentile for $z$ (Table III)

2. $X = \mu + z\sigma$. Draw picture!
Ex 6.5 The top 1% of the IQ distribution.

\[ N(100, 16) \] \( \mu = 100 \), \( \sigma = 16 \). What IQ score must a 62 year-old have to place in the top 1% of the distribution of IQ scores?

99th percentile

For 99th percentile, \( z \) is 2.326.

\[ X = \mu + z \cdot \sigma \]
\[ = 100 + 2.326 \times 16 \]
\[ = 137.216 \]

\[ z = \frac{X - \mu}{\sigma} \quad \Rightarrow \quad X = \mu + z \cdot \sigma \]
Assess Normality

- Look at a histogram or stem-and-leaf plot, check for non-normal features such as gaps, outliers or skewness.
- Compare our observations to the 68-95-99.7 rule for normal distribution:
  - Sample mean \( \bar{x} \)
  - Sample STD \( s \)
  - Whether 68% fall within \([\bar{x} - s, \bar{x} + s]\)
  - 95% \([\bar{x} - 2s, \bar{x} + 2s]\)
  - 99.7% \([\bar{x} - 3s, \bar{x} + 3s]\).

\( \Rightarrow \) normal distribution.