1. Perform the following experiment 20 times. Flip a coin until you get a head OR have 3 tails in a row. Let $X_i$ be the number of times the coin is flipped in the $i$-th experiment. Let $X$ be the sum of the 20 $X_i$’s. Find the mean and variance of $X$.

2. 10 men and 10 women are lined up at random in a row. What is the expected number of men who will have a woman on their right side? Hint: remember that a man might be at the right end of the row with no woman to his right.

3. Two values, $X$ and $Y$, are randomly chosen from the numbers $-1, 0, +1$. Let $W = X+Y$ and $Z = X-Y$. Use the algebra of expectations to compute $\text{Var}(Z)$ and $\text{Cov}(W,Z)$.

4. A die is rolled and then a coin is flipped as many times as the value shown on the die. What is the expected number of heads that appear in this experiment.

5. Prove with moment generating functions that the sum of two independent normal $N(0,1)$ random variables is a normal $N(0,2)$ random variable.

6. A test has a mean score of 80 and a variance of 10. Give a lower bound on the probability of the student’s score being between 75 and 85.

7. In problem 1 above, find the probability that $X$ is greater than 25. Express the answer in terms of $\Phi(\cdot)$

8. An experiment measures the IQ of a person. If historical data indicate that the variance of the IQ distribution is $256 = 16^2$, how many people need to be measured in order that there is a 95% probability that the sample average is within 3 of the true average IQ of the population.

9. Repeat problem #6 but now we look at the average score of three students on the test. Give an estimate with the one-sided Chebychev inequality that their average score is under 85.
1. Perform the following experiment 30 times. Flip a coin 2 times. If the first flip is a head, you win $1 (tail you win nothing). If the second flip is a head, you win $2. Let $X_i$ be your winnings on the i-th experiment. Let $X$ be the sum of the 30 $X_i$. Find the mean and variance of $X$.

2. You want to collect all of the 65 stamps issued by the Post Office this year and place them in the 65 places on the 2006 page of your stamp book. You have collected 200 envelopes, each with one stamp on it that is assumed to be randomly chosen from one of the 65 stamps issued this year. Find the expected number of places for 2006 stamps that are NOT filled in your stamp book after going through the 200 envelopes.

3. Let $X, Y$ be two random variables that are NOT independent. Both have mean 8. Also we know that $E(XY) = 12$. Determine $\text{Cov}(X,Y)$.

4. There are 3 books. The number of (typographical) errors per page of a book is a Poisson random variable. For the first book, $\lambda = 3$; for the second book, $\lambda = 5$, and for the third book, $\lambda = 10$. If a random book is chosen, what is the expected number of errors per page.

5. Prove with moment generating functions that the sum of two exponential random variables with $\lambda = 5$ is a Gamma ($\lambda = 5, n=2$) random variable.

6. The speed of motorists on a particular stretch of highway has a mean speed of 60 and a variance of 16. A motorist’s speed is recorded on this highway. Give a lower bound on the probability of the speed being between 56 and 64 miles per hour.

7. In problem 1 above, find the probability that $X$ is greater than 40. Express the answer in terms of $\Phi(\ )$.

8. An experiment measures how fast a person can learn a particular task. If historical data for similar experiments indicate that the variance of the learning time is 50, how many people need to be tested in order that there is a 95% probability that the sample average of the learning time is within 4 of the true average learning time of the population.

9. Repeat problem #6 but now we look at the average speed of four motorists on the stretch of highway. Give an estimate with the one-sided Chebychev inequality that their average speed is under 65.