Homework 3  
Due Date: November 24th, 2008

1. Pg98, Ex2:  
If \( f \) is continuous mapping of a metric space \( X \) into a metric space \( Y \), prove that  
\[
\bar{f(E)} \subset f(\bar{E})
\]
for every set \( e \subset X \). (\( \bar{E} \) denotes the closure of \( E \).) Show, by an example, that \( f(\bar{E}) \) can be a proper subset of \( f(E) \).

2. Pg98, Ex3: Let \( f \) be a continuous real function on a metric space \( X \). Let \( Z(f) \) (the zero set of \( f \)) be the set of all \( p \in X \) at which \( f(p) = 0 \). Prove that \( Z(f) \) is closed.

3. Pg98, Ex7:  
If \( E \subset X \) and if \( f \) is a function defined on \( X \), the restriction of \( f \) to \( E \) is the function \( g \) whose domain of definition is \( E \), such that \( g(p) = f(p) \) for \( p \in E \). Define \( f \) and \( g \) on \( \mathbb{R}^2 \) by: \( f(0, 0) = g(0, 0) = 0, f(x, y) = xy^2/(x^2 + y^4), g(x, y) = xy^2/(x^2 + y^2) \) if \( (x, y) \neq (0, 0) \).  
Prove that \( f \) is bounded on \( \mathbb{R}^2 \) that \( g \) is unbounded in every neighborhood of \( (0, 0) \), and that \( f \) is not continuous at \( (0, 0) \); nevertheless, the restrictions of both \( f \) and \( g \) to every straight line in \( \mathbb{R}^2 \) are continuous!

4. Pg98, Ex8:  
Let \( f \) be a real uniformly continuous function on the bounded set \( E \) in \( \mathbb{R}^1 \). Prove that \( f \) is bounded on \( E \).  
Show that the conclusion is false if boundedness of \( E \) is omitted from the hypothesis.

5. Pg98, Ex12:  
A uniformly continuous function of a uniformly continuous function is uniformly continuous.  
State this more precisely and prove it.

6. Pg98, Ex14:  
Let \( I = [0, 1] \) be the closed unit interval. Suppose \( f \) is continuous mapping of \( I \) into \( I \).  
Prove that \( f(x) = x \) for at least one \( x \in I \).

7. Pg98, Ex15:  
Call a mapping of \( X \) into \( Y \) open if \( f(V) \) is an open set in \( Y \) whenever \( V \) is an open set in \( X \).  
Prove that every continuous open mapping of \( \mathbb{R}^1 \) into \( \mathbb{R}^1 \) is monotonic.