1. **(20 points)** A ball of mass $m = 4$ is dropped from the top of a building of height $h = 20$. Assume that drag is proportional to velocity with drag coefficient $k = 2$, and that a gravitational force of $g = 10$ accelerates the ball in a downward direction (all in consistent units). ($\ln 2 \approx 0.7$)

   (a) (5 points) Derive the DE for distance of the ball from the top of the building.
   (b) (7.5 points) What is the asymptotic equilibrium velocity of the ball?
   (c) (7.5 points) Does the ball ever achieve a velocity equal to 10?

2. **(20 points)** A homogeneous second-order linear differential equation

   $$x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} - 12y = 0; \quad y(2) = 3 \quad y'(2) = 8$$

   (a) (7.5 points) Show that if $x > 0$ then the substitution $v = \ln x$ transforms the given differential equation into the constant-coefficient linear equation

   $$\frac{d^2 y}{dv^2} + \frac{dy}{dv} - 12y = 0$$

   with independent variable $v$.
   (b) (7.5 points) Find the two linearly independent solutions, $y_1(x)$ and $y_2(x)$, of the given differential equation.
   (c) (5 points) Solve the equation with the given initial conditions.

3. **(20 points)** A nonhomogeneous second-order linear differential equation

   $$y'' + y = 2\sin x$$

   (a) (5 points) Find a complementary function of the given nonhomogeneous equation.
   (b) (10 points) Find a particular solution of the given differential equation using the method of **variation of parameters**.
   (c) (5 points) Find the general solution of the equation.
4. **(20 points)**

A nonhomogeneous third-order linear differential equation

\[ y^{(3)} - y'' - y' + y = 2e^{-x} \]

(a) **(10 points)** Find the three linearly independent solutions \(y_1(x), y_2(x)\) and \(y_3(x)\) of the associated homogeneous differential equation.

(b) **(10 points)** Determine the general solution of the given differential equation.

5. **(20 points)**

Consider the system of two masses and three springs shown in figure. The displacement of the mass \(m_1\) from its static equilibrium position is \(x_1\) and the displacement of the mass \(m_2\) from its static equilibrium position is \(x_2\).

(a) **(3 points)** Derive the equations of motion.

(b) **(2 points)** If \(m_1 = m_2 = 2\), \(k_1 = k_3 = 2\) and \(k_2 = 8\). Write the system of equations in operator notation.

(c) **(5 points)** Eliminate the dependent variable \(x_2(t)\) from the system, and get the equation which can be solved for \(x_1 = x_1(t)\).

(d) **(10 points)** Find the general solution of the system, \((x_1(t), x_2(t))\).