1. Chapter 4.1 Problem 12, Page 251
Solve the following system of equations.
\[
\begin{align*}
x' &= y \\
y' &= x
\end{align*}
\]
Solution: By eliminating \( y \) in first equation gives
\( x'' = y' = x \Rightarrow x'' - x = 0 \)
The characteristic equation of the linear second-order DE is
\( r^2 - 1 = 0 \Rightarrow (r + 1)(r - 1) = 0 \Rightarrow r = -1, r = 1 \)
\( x(t) = Ae^t + Be^{-t} \)
The original first equation
\( y = x' \Rightarrow y(t) = Ae^t - Be^{-t} \)

2. Chapter 4.1 Problem 19, Page 251
Solve the following system of equations with the given initial conditions.
\[
\begin{align*}
x' &= -y \\
y' &= 13x + 4y \\
x(0) &= 0, y(0) = 3
\end{align*}
\]
Solution: Using method of elimination
\( x'' = -13x - 4y = -13x + 4x' \)
\( y'' - 4x' + 13x = 0 \)
The characteristic equation of the linear second-order DE is
\( r^2 - 4r + 13 = 0 \Rightarrow r_1 = \frac{4 + \sqrt{16 - 52}}{2} = 2 + 3i, r_2 = 2 - 3i \)
\( x(t) = -e^{2t} \sin 3t \)
\( y(t) = e^{2t}(3 \cos 3t + 2 \sin 3t) \)

3. Chapter 4.2 Problem 4, Page 262
Use operator method to solve the following system of equation.
\[
\begin{align*}
x' &= 3x - y \\
y' &= 5x - 3y
\end{align*}
\]
(\( x(0) = 1, y(0) = -1 \))
Solution:
\( x(t)(3e^{2t} - e^{-2t})/2 \)
\( y(t) = (3e^{2t} - 5e^{-2t})/2 \)
4. Chapter 4.2 Problem 18, Page 262

Solve the following system of equations.

\[
\begin{align*}
  x' &= x + 2y + z \\
  y' &= 6x - y \\
  z' &= -x - 2y - z
\end{align*}
\]

Solution:

\[
\begin{align*}
  y(t) &= c_1 e^{3t} + c_2 e^{-4t} \\
  x(t) &= (4c_1 e^{3t} - 3c_2 e^{-4t})/6 \\
  z(t) &= (-4c_1 e^{3t} + 3c_2 e^{-4t})/6
\end{align*}
\]

5. Chapter 4.2 Problem 27, Page 262

Use operator method to solve the following system of equations.

\[
\begin{align*}
  (D^2 + 1)x + (D^2 + 2)y &= 2e^{-t} \\
  (D^2 - 1)x + D^2y &= 0
\end{align*}
\]

Solution:

\[
\begin{align*}
  x(t) &= e^{-t} \\
  y(t) &= 0
\end{align*}
\]

6. Chapter 4.2 Problem 38, Page 263

Consider the system of two masses and three springs shown in Fig. 4.2.6. First derive the equations of motion, then solve the second-order linear system for \(m_1 = 1, m_2 = 2, k_1 = 1, k_2 = 2, k_3 = 2\).

\[
\begin{align*}
  m_1x'' &= -(k_1 + k_2)x + k_2y \\
  m_2y'' &= k_2x - (k_2 + k_3)y
\end{align*}
\]

Solution:

\[
\begin{align*}
  x'' &= -3x + 2y \\
  2y'' &= 2x - 3y
\end{align*}
\]

In operator notation:

\[
\begin{align*}
  (D^2 + 3)x - 2y &= 0 \\
  -2x + (2D^2 + 3)y &= 0
\end{align*}
\]

Operational Determinant:

\[
(D^2 + 3)(2D^2 + 3) - 4 = 2D^4 + 9D^2 + 5
\]

DE for \(x\) obtained using Cramer Rule

\[
(2D^4 + 9D^2 + 5)x = 0
\]

DE for \(y\) obtained using Cramer Rule

\[
(2D^4 + 9D^2 + 5)y = 0
\]

Solution of 4th-order DE for \(x\):

\[2x^{iv} + 9x'' + 5x = 0\]

Characteristic equation

\[2r^4 + 9r^2 + 5 = 0 \Rightarrow\]