Exercise 1.2.1
(a) let \( z = r(\cos \theta + i \sin \theta) \)
\[
z^5 = r^5 (\cos 5\theta + i \sin 5\theta) = 2 \Rightarrow r^5 = 2, 5\theta = 2\pi k, \quad k = 0, ..., 4
\]
Thus, \( z = \sqrt[5]{2} \left( \cos \frac{2\pi k}{5} + i \sin \frac{2\pi k}{5} \right) \), \( k = 0, ..., 4 \)
(b) let \( z = r(\cos \theta + i \sin \theta) \)
\[
z^4 = r^4 (\cos 4\theta + i \sin 4\theta) = -i \Rightarrow r = 1, 4\theta = \frac{3\pi}{2} + 2\pi k
\]
Thus, \( z = \cos \left( \frac{3\pi}{8} + \frac{\pi k}{2} \right) + i \sin \left( \frac{3\pi}{8} + \frac{\pi k}{2} \right), \quad k = 0, ..., 3 \)

Exercise 1.2.3
By the definition of the complex conjugate, we have
\[
\overline{(3 + 8i)^4 / (1 + i)^{10}} = \frac{(3 + 8i)^4}{(1 + i)^{10}} = \frac{(3 - 8i)^4}{(1 - i)^{10}}
\]

Exercise 1.2.5
Since
\[
\cos x + i \sin x)^5 = \cos 5x + i \sin 5x
\]
The real and imaginary parts of this formula tell that
\[
\cos 5x = \cos^5 x - 10 \cos^3 x \cdot \sin^2 x + 5 \cos x \cdot \sin^4 x
\]
\[
\sin 5x = \sin^5 x - 10 \cos^2 x \cdot \sin^3 x + 5 \cos^4 x \cdot \sin x
\]

Exercise 1.2.7
\[
\left| \frac{i(2 + 3i)(5 - 2i)}{-2 - i} \right| = \frac{|2 + 3i||5 - 2i|}{|2 + i||2 + i|} = \sqrt{377 / 5}
\]

Exercise 1.2.18
Solution (a) the number \( \bar{z}z \) is nonnegative real, so
\[
\arg(\bar{z}z) = 0
\]
mod \( 2\pi \) and therefore
arg(\(\bar{z}\)) + arg(z) = 0 \mod 2\pi, so
arg(\(\bar{z}\)) = - arg(z) \mod 2\pi.

(b) From the identity \(\frac{z}{w} \cdot w = z\), we get
arg\(\left(\frac{z}{w}\right)\) = arg(z) \mod 2\pi, so
arg\(\left(\frac{z}{w}\right)\) + arg(w) = arg(z) \mod 2\pi, i.e.,
arg\(\left(\frac{z}{w}\right)\) = arg(z) - arg(w) \mod 2\pi.

(c) let \(z = a + bi\), where \(a\) and \(b\) are real. Then \(|z| = 0\) if and only if \(\sqrt{a^2 + b^2} = 0\) which is in turn equivalent to \(a^2 + b^2 = 0\). But \(a^2 \geq 0\) and \(b^2 \geq 0\) since \(a\) and \(b\) are real. We conclude that \(a^2 + b^2 = 0\) holds iff \(a = b = 0\), which is equivalent to \(z = 0\).

Exercise 1.2.29
1 + w + \cdots + w^{n-1} = 0, since
\[(1 - w)(1 + w + \cdots + w^{n-1}) = 1 - w^n\]
Multiple by 1-w and use this equation to show that
\[1 + 2w + \cdots + nw^{n-1} = n/(w - 1)\]

Exercise 1.3.2
(a)
\[e^{3-i} = e^3(e^{-i}) = e^3(\cos 1 - i \sin 1)\]
(b) Using \(\cos z = \frac{1}{2}(e^{iz} + e^{-iz})\), we have
\[\cos(2 + 3i) = \frac{1}{2}\cos 2(e^{-3} + e^3) + \frac{i}{2}\sin 2(e^{-3} - e^3)\]
Exercise 1.3.4

(a) \[ \sin z = \frac{1}{2i} \left( e^{iz} - e^{-iz} \right) = \frac{3}{4} + \frac{i}{4} \Rightarrow e^{iz} = \frac{i + 1}{2} \] or \[ e^{iz} = i - 1 \]

Thus, \( z = \frac{1}{4} \pi + 2\pi n + \frac{i}{2} \log 2 \) and \( z = \frac{3\pi}{4} + 2\pi n - \frac{i}{2} \log 2 \)

(b) \[ \sin z = \frac{1}{2i} \left( e^{iz} - e^{-iz} \right) = 4 \Rightarrow e^{iz} = (4 \pm \sqrt{15})i \]

Thus, \( z = \frac{\pi}{2} \pm \left[ 2\pi n - i \log(4 + \sqrt{15}) \right] \)

Exercise 1.3.6

(a) \[ \log(-i) = \log(|-i|) + i \arg(-i) = \frac{3\pi}{2} i + 2\pi n i \]

(b) \[ \log(1 + i) = \log(|1 + i|) + i \arg(1 + i) = \frac{1}{2} \log 2 + \frac{\pi}{4} + 2\pi n i \]

Exercise 1.3.7

(a) Using (a) in Exercise 1.3.6, we have

\[ (-i)^i = e^{i \log(-i)} = \exp \left( -2\pi \left( n - \frac{1}{4} \right) \right) \]

(b) Using (b) in Exercise 1.3.6, we have

\[ (1 + i)^{1+i} = e^{(1+i) \log(1+i)} = \exp \left( \frac{1}{2} \log 2 - 2\pi n - \frac{\pi}{4} \right) \left[ \cos \left( \frac{1}{2} \log 2 + \frac{\pi}{4} \right) + i \sin \left( \frac{1}{2} \log 2 + \frac{\pi}{4} \right) \right] \]

Exercise 1.3.26

Solution: (a) if \( z = x + iy \), then \( z^2 = x^2 - y^2 + 2ixy \). If \( u + iv = z^2 \), then

\[ u = x^2 - y^2 \quad \text{and} \quad v = 2xy \]

A line parallel to the real axis has the equation \( y = y_0 \). Points on its image satisfy

\[ \frac{u}{y_0^2} = \left( \frac{x}{y_0} \right)^2 - 1 \quad \text{and} \quad \frac{v}{y_0^2} = \frac{2x}{y_0} \]

So that
\[
\frac{u}{y_0} = \frac{v}{2y_0} - 1 \quad \text{and} \quad v^2 - 4y_0^2u - 4y_0^4 = 0
\]

That is,

\[
u = \left(\frac{1}{4y_0^2}\right)v^2 - y_0^2
\]

Which is a parabola.

(b) Take the branch of square root given by

\[
\sqrt{z} = r^{1/2}e^{i\theta / 2}
\]

where \(z = re^{i\theta}\), with \(r \geq 0\) and \(0 \leq \theta \leq 2\pi\). If we write \(\sqrt{z} = u + iv\), it follows that

\[
u = r^{1/2} \cos\left(\frac{\theta}{2}\right) \quad \text{and} \quad v = r^{1/2} \sin\left(\frac{\theta}{2}\right)
\]

Lines parallel to the real axis have the equation \(y = rsin\theta = c\), a constant. Their images satisfy

\[
u v = r \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) = \frac{1}{2} r \sin\theta = \frac{c}{2}
\]

which is the equation of a hyperbola.

**Exercise 1.3.27**

Solution: we take the \(n\)-th power of each of these numbers in the following way. Let \(0 \leq k \leq n - 1\). Then

\[
(w^k)^n = \left(\left(e^{\frac{2\pi i}{n}}\right)^k\right)^n = e^{2\pi ki} = 1,
\]

So indeed these numbers are \(n\)-th roots of unity. Also, \(w^k = w^l\) implies \(e^{2\pi ik/n} = e^{2\pi il/n}\), which implies \(e^{2\pi i (k - l)/n} = 1\), so \(k - l\) is a multiple of \(n\). Thus the numbers \(1, w, w^2, \cdots, w^{n-1}\) are all distinct and so are the desired \(n\) roots.