Lecture 8 Summary (Chapter 4)

- \((A \cup B) = \overline{A \cap B}\), \((\overline{AB}) = \overline{A} \cup \overline{B}\)
  
  From the previous example,
  
  \((A \cup B) = \phi = \overline{A} \cap B\), \((\overline{AB}) = \{HH, TH, TT\} = \overline{A} \cup \overline{B}\)

- \(A\) and \(B\) are mutually exclusive (incompatible) if \(A\) and \(B\) have no outcomes in common.

**Example 1**  Tossing a coin twice

\(A:\) Tail at the second toss \(\implies A = \{HT, TT\}\)

\(D:\) Two heads \(\implies D = \{HH\}\)

\(A\) and \(D\) are mutually exclusive.

2) Counting

(A) Tree diagram

- If the first element or object of an ordered pair can be selected in \(n_1\) ways, and for each of these \(n_1\) ways the second element of the pair can be selected in \(n_2\) ways, then the number of pairs is \(n_1n_2\).

- Suppose a set consists of ordered collections of \(k\) elements and that there are \(n_1\) possible choices for the first element; for each choice of the first element, there are \(n_2\) possible choices of the second element; \ldots; for each possible choice of the first \(k-1\) elements, there are \(n_k\) choices of the \(k\)th element. Then there are \(n_1n_2\cdots n_k\) possible \(k\)-tuples.

**Example 2** A family has just moved to a new city and requires the services of an obstetrician, a pediatrician, a specialist in internal medicine and a general surgeon. If a clinic has four obstetricians, three pediatricians, three specialists in internal medicine and two general surgeons, there are \(n_1n_2n_3n_4 = 4 \cdot 3 \cdot 3 \cdot 2 = 72\) ways to select one doctor of each type such that all doctors practice at this clinic.

**Example 3** If a test consists of 12 true-false questions, there are \(n_1n_2\cdots n_{12} = 2^{12} = 4096\) different ways a student can mark the test paper with one answer to each question.
(B) **Combinations**

The number of possible choices of $r$ objects from a set of $n$ distinct objects:

\[
\binom{n}{r} = \frac{n(n-1) \cdots (n-r+1)}{r(r-1) \cdots 2 \cdot 1}
\]

eg.)

\[
\binom{5}{2} = \frac{5 \cdot 4}{2 \cdot 1} = 10
\]

\[
\binom{15}{4} = \frac{15 \cdot 14 \cdot 13 \cdot 12}{4 \cdot 3 \cdot 2 \cdot 1} = 1365
\]

**Example 4** A committee consists of 10 members. The number of possible choices of two representatives is

\[
\binom{10}{2} = \frac{10 \cdot 9}{2 \cdot 1} = 45.
\]