Lecture 39 Summary (Chapter 10)

2) Small, Independent Samples \((n_1 < 30 \text{ and/or } n_2 < 30)\)

See the review on Chapter 10.

**Example 3** A random sample of 15 alumina ceramic insulators doped in a certain manner yielded a sample average holdoff voltage (KV) of 110 and a sample standard deviation of 24, whereas a random sample of 76 plain alumina ceramic insulators resulted in a sample average of 101 and sample standard deviation of 22. Construct a 95% confidence interval for the difference between the two average voltages.

\[
\begin{align*}
  s_p^2 &= \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(14)(24^2) + (75)(22^2)}{15 + 76 - 2} = 498.5 \\
  \text{df} &= n_1 + n_2 - 2 = 89. \text{ A 95% confidence interval for } \mu_1 - \mu_2 \text{ is }
\end{align*}
\]

\[
\bar{x} - \bar{y} \pm t_{.025} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 110 - 101 \pm (1.96)(22.3) \sqrt{\frac{1}{15} + \frac{1}{76}} = 9 \pm 12.4 = (-3.4, 21.4)
\]

**Example 4** The following table shows summary data on gas pressure on the compression strength of beverage cans and plastic bottles.

<table>
<thead>
<tr>
<th>Beverage</th>
<th>Sample size</th>
<th>Sample mean</th>
<th>Sample SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strawberry drink</td>
<td>15</td>
<td>540</td>
<td>21</td>
</tr>
<tr>
<td>Cola</td>
<td>15</td>
<td>554</td>
<td>15</td>
</tr>
</tbody>
</table>

Do the data suggest that the extra carbonation of cola results in a high average compression strength? Use \(\alpha = .01\).

\(n_1 = n_2 = 15, \bar{x} = 540, \bar{y} = 554, s_1 = 21 \text{ and } s_2 = 15\)

**Step 1:** \(H_0: \mu_1 - \mu_2 = 0 \text{ versus } H_1: \mu_1 - \mu_2 < 0\)

**Step 2:** \(\alpha = .01\)

**Step 3:** Test statistic:

\[
T = \frac{\bar{X} - \bar{Y} - \Delta_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}
\]

Rejection region: \(T \leq -t_\alpha = -t_{.01} = -2.467\) with \(\text{df} = n_1 + n_2 - 2 = 28\).

**Step 4:**

\[
s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(14)(21^2) + (14)(15^2)}{15 + 15 - 2} = 333 \Rightarrow s_p = \sqrt{333} = 18.25
\]

\[
t = \frac{540 - 554}{18.25 \sqrt{\frac{1}{15} + \frac{1}{15}}} = -2.10 > -2.467 \Rightarrow \text{Do not reject } H_0
\]

**Step 5:** \(t_{.025} = 2.048 < 2.10 < 2.467 = t_{.01} \Rightarrow .01 < p\text{-value} < .025\)