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Objectives

• Be able to describe and identify the basic types of factor models.
• Understand how factor models are used to “take apart” returns into their pure effects…and why that’s important.
• Use factor models to better estimate, forecast and manage portfolio risk.
• Apply factor models to problems of performance attribution and measurement.
What’s the Big Idea?
Complex Trajectory
Stellar System

• The star has a proper motion in the galaxy.
• A planet rotates about the star.
• A moon is a satellite of the planet.
Decomposition

- Complex effects can often be reduced to simpler ones.
- A simple model of simple models is easier to understand and use than a single complex one.

The moon’s trajectory can be modeled as a sum of simple motions.
A Good Place to Start…

What Are Factor Models?
Factor Models

- The return of an asset can be represented by a linear combination of a number of factors.
- Systematic factors are shared across different instruments.
- There is also an idiosyncratic factor that is peculiar to each asset.
Capital Asset Pricing Model (CAPM)
Systematic and idiosyncratic effects

\[ r_i(t) - r_f = \beta_i(m(t) - r_f) + \varepsilon_i(t) \]

**CAPM**

*Example*

*single factor*

Excess \( \alpha \) form
Separates an element of performance

\[ r_i(t) - r_f = \alpha_i + \beta_i(m(t) - r_f) + \varepsilon_i(t) \]
Arbitrage Pricing Theory (APT)  
Systematic and idiosyncratic effects  

\[ r_i(t) - r_f = \sum_j \beta_{i,j} (f_j(t) - r_f) + \varepsilon_i(t) \]

Excess \( \alpha \) form  
Separates an element of performance  

\[ r_i(t) - r_f = \alpha_i + \sum_j \beta_{i,j} (f_j(t) - r_f) + \varepsilon_i(t) \]
Model Types

- Risk Factors
- Macroeconomic Influences
- Industry Exposures
- Empirical or Statistical Approaches
- Hybrid Models
Risk Factors

e.g., Fama-French Model

Overall Market

Size Effect

Value Effect

Unique

Stock
Macroeconomic Influences

- Interest Rates
- Steel Costs
- Energy Costs
- Unique

Stock
Industry Exposures

From Left to Right:
- Automotive
- Stock
- Financial Services
- Unique
Empirical & Statistical

- Empirical factors are derived directly from actual return data.
- Estimation approaches are, e.g.,
  - Maximum Likelihood
  - Method of Moments
  - Covariance Matrix Factorization
- Statistically efficient, few parameters; few assumptions
- Complex mathematics; economic insights difficult
Hybrid Models

- Factor models are often based on combinations of risk, macroeconomic and industry factors.
- Most common approach
  - Straightforward incorporation of analysts intuitions
  - Easy to understand results
- Empirical and statistical techniques can often be employed to derive or check for “missing” factors, but this must be done carefully.
Understanding Return
Confounding Effects
example from Jacobs & Levy

- Look at the collective returns of high dividend yield stocks:
  - Are the changes in returns in a collection of high dividend yield stocks due to their yield?
  - The real answer is that you often have no idea.
- What’s the problem?
  - Energy price increases will have a material impact on utilities which also tend to be dividend paying.
  - Thus, conclusions about the behavior of dividend paying stocks will be corrupted by other effects.
Translating Insights into Actions

- **Questions:**
  - Can I take on more risk?
  - Is this industry’s prospects better than that of the general market?
  - Do I want to make a growth or value bet?
  - Do I think this company is more favorably priced than its peers?
- Factor models provide a framework for reviewing the historic effectiveness of a strategy.
- Factors models allows us to turn general insights into specific portfolio allocations.

\[
\begin{align*}
    r_P(t) - r_M(t) &= \alpha_i + \sum_j \left( \beta_{P,j} - \beta_{M,j} \right) (f_j(t) - r_f) + \epsilon_P(t) \\
\end{align*}
\]

- **Overall Return**
- **Systematic Return**
- **Unsystematic Return**
Risk Management
Diversification Effects

- Naïve diversification neglects systematic effects
- Systematic effects can not be diversified away.
Risk Tracking

- Understanding risk, in either absolute or relative terms, is critical.
- Isolating risk exposures provides portfolio-level insights.
Performance Analysis
Factor models describe the underlying stochastic process generating returns.
They provide a coherent and consistent basis for formulating expectations and monitoring performance.

\[ r_P(t) - r_M(t) = \alpha_i + \sum_j (\beta_{P,j} - \beta_{M,j}) (f_j(t) - r_f) + \epsilon_p(t) \]

Overall Return  \rightarrow  Systematic Return

Unsystematic Return
Mean return can be decomposed into separate components.

Manager/company selection can be separated from sector allocation.

\[
\mathbb{E}[r_P - r_M] = \alpha_i + \sum_j (\beta_{P,j} - \beta_{M,j}) \mathbb{E}[(f_j - r_f)]
\]
Portfolio Variance

- The nature of the risks assumed and their relative contribution can be quantified at the portfolio level.
- Avoid the “Everybody’s a genius in a bull market” mistake.

\[
\text{Var}[r_P - r_M] = \sum_j \sum_k (\beta_{P,j} - \beta_{M,j}) (\beta_{P,j} - \beta_{M,j}) \text{Cov}[f_j, f_k] + \text{Var}[\varepsilon_P]
\]
Analyzing Performance

• Factor models give you a common basis for analyzing reward and risk.
• You’ve beaten your benchmark. Did you:
  ▪ Select better companies?
  ▪ Select better industries?
  ▪ Take on more risk?
• You need to compare two companies or investments. Is their difference in performance due to:
  ▪ Manager skill?
  ▪ Market effects?
  ▪ Risk levels?
A Useful Notion…
The Quant’s Trap

Today’s scientists have substituted mathematics for experiments, and they wander off through equation after equation, and eventually build a structure which has no relation to reality.

Nikola Tesla, *Modern Mechanics and Invention* (1934)
Thank You!