1. Verify by substitution that $y_1 = e^{3x}$, $y_2 = e^{-3x}$ are solutions to $y'' = 9y$

Solution: Suppose $y_1 = e^{3x}$. Then $y_1' = 3e^{3x} = 3y_1$ and $y_1'' = 9y_1$. Now suppose $y_2 = e^{-3x}$. Then $y_2' = -3e^{-3x} = -3y_2$ and $y_2'' = 9y_2$

2. Verify by substitution that $y = (x + C) \cos x$ is a solution to $y' + y \tan x = \cos x$.

Solution: Suppose $y = (x + C) \cos x$. Then $y' = \cos x - (x + C) \sin x = \cos - (x + C) \cos x \tan x = \cos x - y \tan x$ \implies $y' + y \tan x = \cos x$

3. Find a function $y$ such that $\frac{dy}{dx} = (x - 2)^2$ and $y(2) = 1$.

Solution: By integration, we see $y(x) = \frac{1}{3}(x - 2)^3 + C$. Since $1 = y(2) = C$, we have $y(x) = \frac{1}{3}(x - 2)^3 + 1$.

4. Find a function $y$ such that $\frac{dy}{dx} = \frac{1}{x^2}$ and $y(1) = 5$

Solution: By integration, we see $y(x) = -\frac{1}{x} + C$. Since $5 = y(1) = C - 1$, we have $y(x) = 6 - \frac{1}{x}$

5. Construct the slope field of $y' = y - x$

Solution:

6. Find the general solution of $y' = 1 + x + y + xy$

Solution: $y' = 1 + x + y + xy = (1 + x)(1 + y) \implies \frac{dy}{1+y} = (1 + x) \, dx \implies \ln |1 + y| = \frac{1}{2} (x + 1)^2 + C \implies y(x) = Ce^{\frac{1}{2}(x+2)^2} - 1$

7. Find the general solution of $x^2 y' = 1 - x^2 + y^2 - x^2 y^2$

Solution: $x^2 y' = 1 - x^2 + y^2 - x^2 y^2 = (1 - x^2)(1 + y^2) \implies \frac{dy}{1+y^2} = \left(\frac{1}{x^2} - 1\right) \, dx \implies \arctan y = C - \frac{1}{x} - x \implies y(x) = \tan\left(C - \frac{1}{x} - x\right)$

8. Find the general solution of $(x^2 + 4) y' + 3xy = x$

Solution: $(x^2 + 4) y' + 3xy = x \implies y' + \frac{3x}{x^2 + 4} y = \frac{x}{x^2 + 4} \implies y' \left(x^2 + 4\right)^{\frac{1}{2}} + \left((x^2 + 4)^{\frac{1}{2}}y\right)' = x(x^2 + 4)^{\frac{1}{2}} \implies \left((x^2 + 4)^{\frac{3}{2}}y\right)' = \frac{1}{3} (x^2 + 4)^{\frac{3}{2}} + C \implies y(x) = \frac{1}{3} + C \left(x^2 + 4\right)^{-\frac{3}{2}}$

9. Find the general solution of $y' - 2xy = e^{2x}$
Solution: \[ y' - 2xy = e^{x^2} \implies e^{-x^2} y' + \left(e^{-x^2}\right)' y = 1 \implies e^{-x^2} y = x + C \implies y(x) = (x + C)e^{x^2} \]

10. Find the general solution of \((x + y)y' = x - y\)

Solution: Suppose \(v = x + y\). Then \(v' = 1 + y', -y = x - v\) and \(x - y = x + x - v = 2x - v\). So \((x + y)y' = x - y \implies v\left(v' - 1\right) = 2x - v \implies vv' = 2x \implies \frac{1}{2}v^2 = x^2 + C \implies \frac{1}{4}(x + y)^2 = x^2 + C\).